Anchoring the Yield Curve Using Survey Expectations[1]

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Abstract

The dynamic behavior of the term structure of interest rates is difficult to replicate with models, and even models with a proven track record of empirical performance have underperformed since the early 2000s. On the other hand, survey expectations can accurately predict yields, but they are typically not available for all maturities and/or forecast horizons. We show how survey expectations can be exploited to improve the accuracy of yield curve forecasts given by a base model. We do so by employing a flexible exponential tilting method that anchors the model forecasts to the survey expectations, and we develop a test to guide the choice of the anchoring points. The method implicitly incorporates into yield curve forecasts any information that survey participants have access to - such as information about the current state of the economy or forward-looking information contained in monetary policy announcements - without the need to explicitly model it. We document that anchoring delivers large and significant gains in forecast accuracy relative to the class of models that are widely adopted by financial and policy institutions for forecasting the term structure of interest rates.

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1 Introduction

The term structure of interest rates contains crucial information for both policymakers' and investors' decisions. Yet, in spite of a vast and growing literature on yield curve modeling, no single approach has emerged that can accurately describe the dynamic behavior of yields. Two popular classes of yield curve models considered in the literature are no-arbitrage dynamic latent factor models (Duffie and Kan, 1996; Litterman et al., 1991; Dai and Singleton, 2000) and the Dynamic Nelson and Siegel (DNS) model of Diebold and Li (2006). These models share a similar state-space structure in which the yields depend on three dynamic latent factors (level, slope, and curvature) extracted from the cross-section of yields, but differ in the restrictions they impose on the model's parameters. Although the latter have become the leading method for yield curve forecasting at many policy institutions (BIS, 2005) due to their successful empirical performance (Diebold and Li, 2006), one of the findings of this paper is that their performance has deteriorated in recent years. The fact that the three-factor structure is not sufficient to capture the dynamics of yields has been documented before (e.g., Diebold and Rudebusch, 2012; Mönch (2008)), and a general consensus has emerged in the literature that one must look beyond the cross-section of yields to pin down the dynamic behavior of interest rates, for example, by enlarging the model's information set with either observable macroeconomic factors (Diebold et al., 2006; Ang and Piazzesi, 2003; Hördahl et al., 2006; Rudebusch and Wu, 2008; Mönch, 2008; Coroneo et al., 2016) or latent "hidden" factors (Joslin et al., 2010; Duffee, 2011). A thorough review of the literature on the connections between analysis of the term structure and the macroeconomics is Gürkaynak and Wright (2012).

This paper's premise is that latent factor models neglect a key determinant of yield dynamics: expectations about future economic developments. It is a well-documented fact that expectations contained in survey data can accurately forecast key macroeconomic variables, such as GDP, inflation, and yields, especially at short forecast horizons (Stark, 2010; Chun, 2012), and several recent papers have utilized survey data in the analysis of the term structure of interest rates. For example, Chun (2011) uses Blue Chip Financial Analysts (henceforth BC) forecasts as observable factors in a no-arbitrage dynamic latent factor model; Chernov and Mueller (2012) develop a model that incorporates survey expectations and links them to the "hidden factor" of Joslin et al. (2010) and Duffee (2011); Dijk et al. (2014) use (longer-horizon) survey expectations to improve estimates of some parameters in the DNS model, and Kim and Orphanides (2012) use survey data to overcome some small-sample estimation problems in no-arbitrage dynamic latent factor models.

Rather than incorporating survey data into the model, we employ a formal method that anchors segments of the yield curve forecasts to the corresponding survey expectations and transmits the superior forecasting ability to the rest of the yield curve. In essence, the anchoring constrains the dynamics of some yields to replicate those of the survey expectations and thus implicitly incorporates into the forecasts of the whole yield curve any information that survey participants have access to, without the need to explicitly model it. This can include information about the current state of the economy that survey participants deem relevant for predicting future interest rates and that they potentially extract from large dimensional data sets. In this respect, survey expectations offer the possibility to capture both observable and "hidden" factors that can explain yield curve dynamics (as also argued by Duffee, 2011). Survey expectations can also reflect additional useful information, such as nonlinearities (for example, the zero-lower bound constraint), structural change, and information about the future course of monetary policy that may be difficult to capture with existing backward-looking models. In this paper, we stress in particular the role played by the ability of survey participants to capture the kind of forward-looking information about interest rates that is increasingly contained in monetary policy announcements.

An important question we address is which segments of the yield curve one should anchor, as one typically has access to survey expectations about a subset of points along the yield curve. Our main result is to derive a testable condition such that anchoring delivers an improvement in *density forecast* accuracy for the whole yield curve. In our data, we found that the largest improvement is offered by anchoring using the 3-month survey forecast. As a quick visualization of the effects of anchoring, consider Figure 5, which shows that the method shifts the yield curve forecast toward the actual realization, resulting in sizable accuracy improvements that are particularly visible in regions of the yield curve near the anchoring point.

The theoretical justification of the method is based on exponential tilting (see Robertson et al., 2005 and Giacomini and Ragusa, 2013). The testable condition to guide the choice of anchoring points is a new result.

We apply the method to incorporate Blue Chip financial analysts' monthly expectations about yields into yield curve forecasts based on the DNS model. It is worth emphasizing that, although we take the DNS model as a benchmark due to its popularity in the forecasting literature, the anchoring method is more generally valid and could be applied to any alternative model of the yield curve.

We find that the anchoring procedure results in forecasts that significantly outperform those from the base model. The accuracy gains are sizable, averaging about 30% and up to 52%. The anchored forecast is also the only one that was able to beat the random walk over the period 2000-2011. These results are robust to considering a subsample that ends in 2008, which suggests that the good performance of our method is not solely driven by the fact that survey participants correctly incorporate zero lower bound constraints. This is also the reason why we don't consider in our comparison models that explicitly take the zero lower bound constraint into account (for an interesting recent example, see Christensen and Rudebusch, 2015).

Although these improvements are important on their own, we provide further insight into the economic forces driving the superior performance of the anchored forecasts. We find that the anchored forecasts implicitly incorporate measures of real activity and forward-looking information contained in monetary policy announcements. The ability of the anchoring method to incorporate the information contained in monetary policy announcements, in particular, has two important implications. The first is that the anchoring method is likely to become even more useful as a practical tool for forecasters and central bankers in the future, now that forward guidance has been formally adopted by several central banks around the world, including the Federal Reserve, the Bank of England and the ECB. The second is that any successful attempt to explicitly model the dynamics of yields should acknowledge the value of forward-looking information.

The paper is organized as follows. Section 2 reviews the DNS model. Section 3 describes the anchoring method and derives the test for choosing the anchoring points. Section 4 contains the empirical results and Section 5 concludes. An online appendix contains a description of the data, in-sample estimation results of the DNS model, and the a comparison of our method and the conditional forecast.

2 The DNS model and its variants

The DNS model introduced by Diebold and Li (2006) for an m-dimensional vector of yields y_t with typical element $y_t(\tau)$, where τ is the maturity, is given by:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) + u_t(\tau), \tag{1}$$

where the dynamic factors β_{1t}, β_{2t}, and β_{3t} are interpreted as the level, slope, and the curvature of the yield curve and λ is a fixed parameter governing the exponential decay rate of the first and second component in (1). Following Diebold and Li (2006), we let $\lambda = 0.069$, the value that maximizes the loading on the curvature factor for the yields with maturity 30 months.

When we observe a series of yields $y_t(\tau_i)$, for a set of n maturities, $\tau_1 < \tau_2 < \ldots < \tau_n$, the yield curve can be estimated by cross sectional regressions. From these regressions we obtain a series of estimated factors $\left\{\hat{\beta}_{1t}, \hat{\beta}_{2t}, \hat{\beta}_{3t}\right\}_{t\leqslant T}$ which can themselves be modeled dynamically to provide out-of-sample forecasts of the factors and thus of the entire yield curve. The main drawbacks of this two-step approach are, first, that the uncertainty associated with the estimation of the factors is not acknowledged in the second step when the yields are calculated, and, second, the loss of efficiency arising from not exploiting the correlation structure.

To mitigate these problems, we follow Diebold et al. (2006) and exploit the state-space representation of the DNS model. To describe this representation, rewrite equation (1) by stacking the yields for the n maturities

$$y_t = \Gamma_y \beta_t + u_t^y, \tag{2}$$

where $y_t = (y_t(\tau_1), \ldots, y_t(\tau_n))'$, $\beta_t = (\beta_{1t}, \beta_{2t}, \beta_{3t})'$, and Γ is a $n \times 3$ matrix of loadings whose $i-th$ row is equal to $\left(1, \left(1-e^{-\lambda\tau_i}\right)/\lambda\tau_i, \left(1-e^{-\lambda\tau_i}\right)/\lambda\tau_i - e^{-\lambda\tau}\right)$. The disturbance vector is given by $u_t^y = (u_t(\tau_1), \ldots, u_t(\tau_n))'$, $u_t^y \sim \mathcal{N}(0, Q)$. The time series process for β_t is a vector autoregression:

$$\beta_{t+1} = \bar{\beta} + \Phi_y \beta_t + \xi_t, \quad \xi_t \sim \mathcal{N}(0, R), \tag{3}$$

where $\bar{\beta}$ is a (3×1) vector and Φ_y is a (3×3) matrix. Equations (2) and (3) constitute the basic version of the model for the yield curve we consider here. We will refer to this model as the "yield only" model to distinguish it from its extension in which the dynamics of yields and macroeconomic variables are modeled in terms of common factors, as recently proposed by Coroneo et al. (2016). We call this model the "macro augmented" DNS model.

Let x_t be a $(p \times 1)$ vector of macroeconomic variables. The macro augmented DNS model consists of a state-space model with measurement equation

$$\begin{pmatrix} y_t \\ x_t \end{pmatrix} = \begin{pmatrix} 0 \\ \bar{x} \end{pmatrix} + \begin{bmatrix} \Gamma_y & 0 \\ \Gamma_{xy} & \Gamma_x \end{bmatrix} \begin{pmatrix} \beta_t \\ \theta_t \end{pmatrix} + \begin{pmatrix} u_t^y \\ u_t^x \end{pmatrix}, \tag{4}$$

and transition equation

$$\begin{pmatrix} \beta_{t+1} \\ \theta_{t+1} \end{pmatrix} = \begin{pmatrix} \bar{\beta} \\ \bar{\theta} \end{pmatrix} + \begin{bmatrix} \Phi_y & \Phi_{yx} \\ \Phi_{xy} & \Phi_x \end{bmatrix} \begin{pmatrix} \beta_t \\ \theta_t \end{pmatrix} + \begin{pmatrix} \xi_t^y \\ \xi_t^x \end{pmatrix}. \tag{5}$$

The dimension of Γ_x depends on the number of common factors that determine the dynamics of the macro variables. Several studies have shown that two factors, one real and one nominal, are sufficient to capture the dynamics of a large variety of macroeconomic indicators for the US Sargent et al. (1977); Giannone et al. (2005). We accordingly choose two factors so that the dimension of Γ_x is $(p \times 2)$.[1]

The idiosyncratic disturbances, $u_t \equiv (u_t^{y'}, u_t^{x'})'$, and the transition equation disturbances, $\xi_t \equiv (\xi_t^{y'}, \xi_t^{x'})'$, are assumed to be Gaussian

$$\begin{pmatrix} u_t^y \\ u_t^x \end{pmatrix} \sim \mathcal{N}\left(0, \begin{bmatrix} Q_y & 0 \\ 0 & Q_x \end{bmatrix}\right), \begin{pmatrix} \xi_t^y \\ \xi_t^x \end{pmatrix} \sim \mathcal{N}\left(0, \begin{bmatrix} R_y & R_{yx} \\ R_{xy} & R_x \end{bmatrix}\right). \tag{6}$$

Both Q_y and Q_x are diagonal, while the covariance of ξ_t is left unrestricted. The assumption that the variance of the idiosyncratic disturbances is diagonal, which implies that the deviations of yields of various maturities from the yield curve are uncorrelated, is quite standard Diebold et al. (2006). This assumption also gives computational tractability given the large number of yields and macro factors we use.[2] The processes u_t and ξ_t are also assumed to be independent, and thus β_t and θ_t are not allowed to react to shocks to y_t and x_t.

[1] Coroneo et al., 2016 consider a similar model to ours using different data and a different sample period and also find that two factors capture well the dynamics of macro economic variables for the US.

[2] Coroneo et al. (2016) relax this restriction by allowing the idiosyncratic errors to follow a first order autoregressive process. However, they restrict the variance of this process to be fixed in an arbitrary way.

We perform estimation by maximum likelihood using the Expectation Maximization (EM) algorithm, which requires at each iteration only one run of the Kalman smoother. As reported in Doz et al. (2012), this estimation technique is feasible when the number of variables is large, and is robust to non Gaussianity and to cross-sectional correlation in u_t.

Density forecasts of yields at different maturities can be obtained as

$$y_{t+h}|y_t,\ldots,y_1,x_1,\ldots,x_t \sim \mathscr{N}(\hat{\mu}_{t+h},\hat{\Sigma}_{t+h}),\ \ h=1,\ldots,H, \tag{7}$$

with $\hat{\mu}_{t+h}=\hat{\Gamma}_y E[\beta_{t+h}|y_t,\ldots,y_1,x_1,\ldots,x_t]$, $\hat{\Sigma}_{t+h}=\hat{\Gamma}_y \mathrm{Var}[\beta_{t+h}|y_t,\ldots,y_1,x_1,\ldots,x_t]\hat{\Gamma}_y'+\hat{Q}_y$, where the conditional mean and the conditional variance of β_{t+h} are those from the Kalman recursions with missing observations from $t+1$ to $t+h$. We will refer to the pdf of y_{t+h} conditional on the information available at time t as $f_t(y_{t+h})$.

3 The anchoring method

In this section, we illustrate the anchoring method for incorporating the information contained in survey expectations into an existing model-based forecast. We then discuss how to choose the anchoring points.

The method is presented without reference to a specific forecasting model, as it can be applied to any model that provides a density forecast. We make the simplifying assumption that the sequence of h-step-ahead density forecasts for the vector of yields is normal with (conditional) mean $\hat{\mu}_{t+h}$ and variance $\hat{\Sigma}_{t+h}$,[3]

$$y_{t+h}|y_t,\ldots,y_1,x_t,\ldots,x_1 \sim \mathscr{N}(\hat{\mu}_{t+h},\hat{\Sigma}_{t+h}),\ \ h=1,\ldots,H,\ t=1,\ldots,T. \tag{8}$$

At time t, we observe the h-step ahead survey forecast for yields for the first $r<m$ maturities $(\tau_1,\ldots,\tau_r)$, that we denote as $\tilde{\mu}_{t+h,1:r}$.[4] Let $y_{t,1:r}$ denote the $r\times 1$ sub-vector of y_t containing yields for maturities $(\tau_1,\tau_2,\ldots,\tau_r)$.

We approach the problem of incorporating $\tilde{\mu}_{t+h,1:r}$ into the forecast from an information theoretic point of view, by projecting the density forecast f_t onto the space of densities that

[3]See Giacomini and Ragusa (2013) for the general case of a nonnormal density forecast.

[4]In the interest of clarity, we consider only the case in which the survey forecasts considered are for maturities $\tau_1,\tau_2,\ldots,\tau_r$. It is immediate to extend the results to the case of non-contiguous maturities.

have conditional mean equal to the survey forecasts for maturities $\tau_1, \dots, \tau_r$. More formally, this set of densities can be characterized as

$$\widetilde{\mathcal{H}}_{t+h} = \left\{ h_t : \int y_{t+h,1:r} h_t(y_{t+h}) dy_{t+h} = \tilde{\mu}_{t+h,1:r} \right\}. \tag{9}$$

It is important to note that no constraints are imposed on the forecasts of yields at longer maturities, $\tau_{r+1}, \dots, \tau_m$. The idea is to select the density in $\widetilde{\mathcal{H}}_{t+h}$ that is "closest" to the model-based density forecast f_t, where closeness is measured by the Kullback-Leibler information criterion:

$$h_t^*(y_{t+h}) = \arg \min_{h \in \widetilde{\mathcal{H}}_{t+h}} \int \log \left(\frac{h_t(u)}{f_t(u)} \right) h_t(u) du. \tag{10}$$

Minimization problems such as (10) play an important role in statistics and econometrics (Csiszár, 1975; White, 1982; Kitamura and Stutzer, 1997; Newey and Smith, 2004; Ragusa, 2011), and they have been considered in the forecasting literature by Robertson et al. (2005) and Giacomini and Ragusa (2013).[5] Any of the previous references show that the solution is a new multivariate density:

$$h_t^*(y_{t+h}) = \exp \left\{ \zeta_t + \xi_t' \left[y_{t+h,1:r} - \tilde{\mu}_{t+h,1:r} \right] \right\} f_t(y_{t+h}), \tag{11}$$

where ζ_t and ξ_t are parameters chosen in such a way that $h_t^*(y_{t+h}) \in \widetilde{\mathcal{H}}_{t+h}$. For the special case of a base density that is multivariate normal, Giacomini and Ragusa (2013) show that we have the following analytical expression for $h_t^*(y_{t+h})$:

$$h_t^*(y_{t+h}) = (2\pi)^{-\frac{m}{2}} \left| \hat{\Sigma}_{t+h} \right|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (y_{t+h} - \mu_{t+h}^*)' \hat{\Sigma}_{t+h}^{-1} (y_{t+h} - \mu_{t+h}^*) \right\}, \tag{12}$$

with

$$\mu_{t+h}^* = \begin{pmatrix} \tilde{\mu}_{t+h,1:r} \\ \hat{\mu}_{t+h,r+1:m} - \hat{\Sigma}_{t+h,21} \left(\hat{\Sigma}_{t+h,11} \right)^{-1} \left(\hat{\mu}_{t+h,1:r} - \tilde{\mu}_{t+h,1:r} \right) \end{pmatrix} \tag{13}$$

[5] A more technical and general discussion of the existence of a solution to these class of problems is given in Komunjer and Ragusa (2016).

and $\hat{\Sigma}_{t+h,11}$ and $\hat{\Sigma}_{t+h,21}$ are blocks of the partitioned matrix $\hat{\Sigma}_{t+h}$:

$$\hat{\Sigma}_{t+h} = \begin{pmatrix} \underset{r\times r}{\hat{\Sigma}_{t+h,11}} & \underset{r\times(m-r)}{\hat{\Sigma}_{t+h,12}} \\ \underset{(m-r)\times r}{\hat{\Sigma}_{t+h,21}} & \underset{(m-r)\times(m-r)}{\hat{\Sigma}_{t+h,22}} \end{pmatrix}. \tag{14}$$

Thus, the solution to (10) is a normal density with the same variance as the initial forecast density, $\hat{\Sigma}_{t+h}$, but a mean that is equal to the survey forecast for those yields that are directly restricted, and for the remaining yields it is equal to a combination between the model forecast and the discrepancy between the survey and the restricted model forecasts. The effect of anchoring the first r yields to the survey forecasts on the other yields depends on this discrepancy and on $\hat{\Sigma}_{t+h}$. Since forecasts of yields at different maturities are generally positively correlated, this implies that when the model forecast is larger (smaller) than the survey forecast $\mu_{t+h,r+1:N}$ is adjusted downwards (upwards).

3.1 Where to anchor?

A natural question is which survey forecasts one should use to anchor the yield curve, given that survey expectations could be available for a number of yields. Proposition 1 derives testable conditions that provide guidance on where to anchor the yield curve. The sufficient condition in Proposition 1 has the intuitive interpretation that anchoring delivers accuracy improvements for the whole density forecast if the survey forecast contains additional information relative to the model-based forecast (for a single yield, the condition is equivalent to a normalized version of the null hypothesis of a forecast encompassing test for the survey relative to the model forecast, see e.g., Clark and McCracken, 2001, pg. 90).

Let $\hat{e}_{t+h,1:r} = y_{t+h,1:r} - \hat{\mu}_{t+h,1:r}$ and $\tilde{e}_{t+h,1:r} = y_{t+h,1:r} - \tilde{\mu}_{t+h,1:r}$ denote the $r\times 1$ vectors containing the model- and survey-based $h-$step-ahead forecasts for yields with maturities $(\tau_1, \tau_2, \ldots, \tau_r)$, respectively.

Proposition 1. *A sufficient condition for the anchored density forecast $h_t^*(y_{t+h})$ to be more accurate than the base forecast $f_t(y_{t+h})$ according to the logarithmic scoring rule, i.e.,*

$$E\left[\log\left(\frac{h_t^*(y_{t+h})}{f_t(y_{t+h})}\right)\right] > 0, \tag{15}$$

is that

$$E\left[S_t\right] \geqslant 0, \tag{16}$$

where $S_t := (\hat{e}_{t+h,1:r} - \tilde{e}_{t+h,1:r})' \left(\hat{\Sigma}_{t+h,11}\right)^{-1} \tilde{e}_{t+h,1:r}$. *A sufficient and necessary condition is that*

$$E\left[S_t + N_t\right] \geqslant 0, \tag{17}$$

where $N_t := \frac{1}{2}(\hat{e}_{t+h,1:r} - \tilde{e}_{t+h,1:r})' \left(\hat{\Sigma}_{t+h,11}\right)^{-1} (\hat{e}_{t+h,1:r} - \tilde{e}_{t+h,1:r})$.

Proof. Since

$$E\left[\log\left(\frac{h_t^*(y_{t+h})}{f_t(y_{t+h})}\right)\right] = E\left[\zeta_t + \xi_t \tilde{e}_{t+h}(\tau)\right], \tag{18}$$

it is sufficient to establish that the expectations of both terms on the right-hand side of the equation are positive to establish that condition (17) is necessary and sufficient. To this end, we show that $\zeta_t = N_t$ and $E\left[\xi_t' \tilde{e}_{t+h,1:r}\right] = E[S_t]$. Since N_t is a.s. positive its expected value is also positive. Thus, positiveness of $E[S_t]$ is sufficient for $E\left[\zeta_t + \xi_t \tilde{e}_{t+h}(\tau)\right] \geqslant 0$.

Let J be the $(r \times m)$ matrix selecting the element of y_{t+h} and μ_{t+h}^* corresponding to maturity $(\tau_1, \tau_2, \ldots, \tau_r)$, i.e., $Jy_{t+h} = y_{t+h,1:r}$. The density $h_t^*(y_{t+h}) = \exp\left\{\zeta_t + \xi_t'\left[Jy_{t+h} - J\mu_{t+h}^*\right]\right\} f_t(y_{t+h})$ is given by

$$\begin{aligned} h_t^*(y_{t+h}) = {} & (2\pi)^{-\frac{m}{2}} \left|\hat{\Sigma}_{t+h}\right|^{-\frac{1}{2}} \\ & \times \exp\left\{-\frac{1}{2}(y_{t+h} - \hat{\mu}_{t+h})'\hat{\Sigma}_{t+h}^{-1}(y_{t+h} - \hat{\mu}_{t+h}) + \zeta_t + \xi_t'\left[Jy_{t+h} - J\mu_{t+h}^*\right]\right\}. \end{aligned}$$

The term inside the exponential can be written as

$$\begin{aligned} & -\frac{1}{2}(y_{t+h} - \hat{\mu}_{t+h})'\hat{\Sigma}_{t+h}^{-1}(y_{t+h} - \hat{\mu}_{t+h}) + \zeta_t + \xi_t'\left[Jy_{t+h} - J\mu_{t+h}^*\right] \\ = {} & -\frac{1}{2}\left(y_{t+h}'\hat{\Sigma}_{t+h}^{-1}y_{t+h} - y_{t+h}'b - b'y_{t+h}\right) + c, \end{aligned}$$

where $b = J'\xi + \hat{\Sigma}_{t+h}^{-1}\hat{\mu}_{t+h}$ and $c = \zeta_t - \xi_t' J\mu_{t+h}^* - \frac{1}{2}\hat{\mu}'\hat{\Sigma}_{t+h}^{-1}\hat{\mu}_{t+h}$. Completing the squares by adding and subtracting $b'\hat{\Sigma}_{t+h}b$ yields $-\frac{1}{2}\left(y_{t+h}'\hat{\Sigma}_{t+h}^{-1}y_{t+h} - y_{t+h}'b - b'y_{t+h} + b'\hat{\Sigma}b\right) + c + \frac{1}{2}b'\hat{\Sigma}_{t+h}b$.

Substituting this expression in the expression for $h_t^*(y_{t+h})$ gives

$$h_t^*(y_{t+h}) = (2\pi)^{-\frac{m}{2}} \left|\hat{\Sigma}_{t+h}\right|^{-\frac{1}{2}} \exp\left(c + \tfrac{1}{2}b'\hat{\Sigma}_{t+h}b\right) \exp\left\{-\frac{1}{2}\left(y_{t+h} - \hat{\Sigma}_{t+h}b\right)' \hat{\Sigma}_{t+h}^{-1}\left(y'_{t+h} - \hat{\Sigma}_{t+h}b\right)\right\}. \tag{19}$$

Imposing the constraint $\int Jy_{t+h}h^*(y_{t+h})dy_{t+h} = J\mu^*_{t+h}$ implies that $J\hat{\Sigma}_{t+h}b{=}J\mu^*_{t+h}$. Solving this last equation for ξ_t gives $\xi_t = \left(J\hat{\Sigma}_{t+h,11}J'\right)^{-1}(J\mu^*_{t+h} - J\hat{\mu}_{t+h})$. To obtain the expression for ζ_t, note that we must have that $c + \frac{1}{2}b'\hat{\Sigma}_{t+h}b = 0$. Solving this restriction for ζ_t yields, after tedious yet straightforward manipulations, $\zeta_t = \frac{1}{2}(J\mu^*_{t+h} - J\hat{\mu}_{t+h})'\left(J\hat{\Sigma}_{t+h}J'\right)^{-1}(J\mu^*_{t+h} - J\hat{\mu}_{t+h})$. Noting that $(J\mu^*_{t+h} - J\hat{\mu}_{t+h}) = (J\mu^*_{t+h} - Jy_{t+h} - J\hat{\mu}_{t+h} + Jy_{t+h}) = (\hat{e}_{t+h,1:r} - \tilde{e}_{t+h,1:r})$ and that $\left(J\hat{\Sigma}_{t+h}J'\right)^{-1} = \left(\hat{\Sigma}_{t+h,11}\right)^{-1}$ gives the desired result. □

The necessary and sufficient condition (17) can be empirically tested using a modification of Giacomini and Rossi (2010)'s fluctuation test, which accounts for the possibility that the expectation might be changing over time. For ease of exposition, in the following we omit the reference to the forecast horizon h, with the understanding that the size of the out-of-sample period T will be different for different forecast horizons. The test takes as primitives two sequences of out-of-sample forecast errors for the survey forecast and for the model-based forecast, $\tilde{e}_t$ and $\hat{e}_t$ for $t = 1, ..., T$. A test of the null hypothesis that anchoring gives a more accurate forecasts according to the logarithmic score can be obtained by letting $\Delta L_t = S_t + N_t$ in the fluctuation test. The test is implemented by choosing a fraction δ of the total out-of-sample size T and computing a sequence of standardized rolling means of ΔL_t:

$$F_{t,\delta} = \hat{\sigma}^{-1}(\delta T)^{-1/2} \sum_{j=t-\delta T+1}^{t} \Delta L_t, \quad t = \delta T, ..., T, \tag{20}$$

where $\hat{\sigma}$ is an HAC estimator of the standard deviation of ΔL_t computed over the rolling window, typically with truncation lag $h-1$, where h is the forecast horizon. The null hypothesis $H_0 : E[\Delta L_t] \geqslant 0$ is rejected when $\min_{t \leq T} F_{t,\delta} < -k_{\delta,\alpha}$, where the critical value $k_{\delta,\alpha}$ is given in Table 1.

The statistic is the empirical version of the logarithmic scoring rule evaluated on a rolling window of observations. As such, its magnitude sheds light on the relative performance of the anchored forecast versus the baseline forecast: positive values of $F_{t,\delta}$ occur in windows where anchoring improves the accuracy of the density forecast.

[Table 1 about here.]

4 Empirical results

4.1 The deteriorating performance of the DNS model and its variants

In this section, we document how the forecasting performance of the DNS model has deteriorated in the years after those considered by Diebold and Li (2006), who found that the model performed well in the sample from 1985-2000. This has been noted before, for example, by Diebold and Rudebusch (2012) and Mönch (2008). We complement their results by showing that the same occurs when augmenting the DNS model to incorporate information extracted from macroeconomic data.

For the yield-only model, we estimate the DNS models using the series of U.S. zero-coupon yields constructed in Gürkaynak et al. (2007).[6] We consider average-of-the-month data from January 1985 to December 2011 on yields with the following maturities expressed in months: 6, 9, 12, 15, 18, 21, 24, 30, 36, 48, 60, 72, 84, 96, 108, 120. We augment the yield data with the monthly time series of the 3-month Treasury constant maturity rate from the FRED data set (code GS3M), which corresponds to the rate forecasted by the BC analysts.[7] In total we have a panel of 324 monthly observations on 17 yields.

We estimate the DNS models using an out-of-sample recursive scheme and consider forecast horizons of 3-, 6-, 9- and 12-months ahead. The first estimation period uses data from 1985:1 to 1999:12, and we evaluate the forecasts over the out-of-sample period 2000:1 to 2011:12. We compare the mean squared forecast error (MSFE) of each variant of the DNS model to that of a random walk benchmark, which forecasts the yields as $\hat{\mu}_{t+h} = y_t$. The MSFE for the forecast of a yield of maturity τ at horizon h is given by:

$$MSFE_h(\tau) = \frac{1}{T}\sum_t \left(\hat{\mu}_{t+h}(\tau) - y_{t+h}(\tau)\right)^2 , \tag{21}$$

where T is the size of the out-of-sample portion of the sample, which in our case is $T = 144 - h$.

[6] A detailed description of the data is given in the online appendi. We also performed a similar exercise using the Fama-Bliss data (from CRSP), which are only available for one- to five-year maturities, and obtained similar conclusions, which we do not report in the paper.

[7] We also conducted the analysis using end-of-the-month data and the 3-month yield from the Gürkaynak et al., 2007 data set and obtained qualitatively similar results, which we do not report in the paper.

[Figure 1 about here.]

Figure 1 shows that the random walk outperforms both versions of the DNS model. This is generally true for all maturities and all forecast horizons, with a particularly poor performance for maturities around five years. The only exception appears to be the 10-year yield, for which the model performs as well as the random walk except for the the three-month horizon. An implication is that in our sample incorporating macroeconomic information into the DNS model does not improve its performance. Actually, if anything, the performance of the richer model is poorer especially at shorter maturities.

We should point out that the poor out-of-sample performance of the DNS model in recent years stands in contrast to its good in-sample performance, which we document in the online appendix.

4.2 Testing the condition in Proposition 1

We test the condition in Proposition 1 in order to understand which survey forecasts would deliver the best improvements when used as anchoring points. The BC survey forecasts of yields are available for maturities of 3, 6, 12, 24, 60, and 120 months and for forecast horizons of 3, 6, 9, and 12 months.

Figure 2 reports the results for the test of the condition in Proposition 1, separately considering each maturity and each horizon. The null hypothesis is rejected when the sequence of test statistics crosses the horizontal (red) solid line, which represents the critical value. Although the null hypothesis is not rejected for several survey forecasts, the 3-month yield survey forecast is the one offering the largest improvements, and it is also the yield for which the value of the test statistics is always positive at all horizons. This suggests that, if one were to anchor to a single survey forecast, the 3-month yield forecast would be the best in terms of density forecast performance. The 6-month yield survey forecast also delivers improvements in density forecast performance, but they are smaller than when using the 3-month. A natural question to ask is whether anchoring using both 3-month and 6-month survey forecasts would give additional improvements. In Figure 3, we thus report the test statistics for the test of the condition in Proposition 1 when anchoring using both survey forecasts. While the null hypothesis is again not rejected, the statistic is much smaller than the one using only the 3-month, suggesting that part of the information contained in the 6-month survey forecast overlaps with that of the 3-month, so that when they are used in combination there is practically no advantage.

[Figure 2 about here.]

[Figure 3 about here.]

One of the possible explanations for why the survey forecast of the 3-month yield offers informational advantages over the model-based forecast is that this rate closely reacts to macroeconomic news, such as monetary policy decisions. This information gap between surveys and models is likely to be particularly large when the economic environment is changing quickly, making it more difficult for an econometric model to incorporate the new information. This conjecture is corroborated by considering how model-based and survey-based forecasts respond to monetary policy announcements that contain explicit reference to the likely future path of the short-term rate. There have been several instances of monetary policy statements containing forward-looking information of this kind in recent years, especially since the Federal Reserve began adopting forward guidance as a policy measure. Figure 4 below shows how the survey- and model-based forecasts reacted to one particular episode of forward guidance. The figure reports the 1- to 4-quarter-ahead forecasts of the 3-month yield given by the model and the surveys before and after the FOMC Statement of August 9, 2011, which stated that the "Committee currently anticipates that economic conditions [....] are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013". The figure clearly shows that before the announcement both the model and the survey participants predicted a rate increase for the following year. However, after the announcement the surveys immediately incorporated the information about the policy decision to keep the rate fixed, whereas the model continued to predict a rate hike for several months afterwards, an increase that didn't materialize. The ability to quickly incorporate this information gave the survey forecast a clear advantage, and this is likely to have occurred on several other occasions during the period that we considered, which was characterized by several episodes of forward guidance.

[Figure 4 about here.]

It is worth emphasizing that the informational advantage of the survey expectations is not due to a misalignment of the information sets on which the survey and the model forecasts are based. As we explain in the online appendix, we were careful in matching the timing of the two forecasts. In the online appendix we also explain how we transformed the quarterly BC forecasts into monthly forecasts.

4.3 Anchoring improves accuracy

In this section, we apply the anchoring method described in Section 3 to the DNS model using as anchor the 3-month yield BC forecast, which was the survey forecast offering the largest improvements in density forecast accuracy, according to the test of the condition in Proposition 1. Here we assess the out-of-sample point-forecast performance of the individual yield forecasts relative to the DNS forecasts and to the random walk benchmark.

Table 2 reports relative MSFE for the anchored forecasts against either the forecasts from the base DNS model or the random walk benchmark, for each maturity and forecast horizon. Table 3 reports the same results for a restricted sample that ends in 2008 and thus excludes the zero lower bound period. The asterisks indicate that the Diebold and Mariano (1995) test rejects the null of equal forecast accuracy at 10% against the alternative that the anchored forecast is more accurate. The table shows that the anchored forecasts significantly and strongly outperform the DNS forecasts for almost all maturities and forecast horizons, with typical forecast accuracy gains of about 30% and up to 52%. The only exception is for a few long maturities and short forecast horizons, for which the anchored forecasts and the DNS forecasts perform equally well. The table also shows that the anchored forecasts outperform the random walk, and significantly for maturities up to 15 months and forecast horizons up to 6 months ahead, in a sample in which the DNS model and its variants consistently failed (as shown in Figure 1). These conclusions are robust to excluding the zero lower bound period, suggesting that the superior performance of our method is not solely driven by the ability of the survey expectations to incorporate the zero lower bound constraint.

[Table 2 about here.]

[Table 3 about here.]

Figure 5 reports the yield curve implied by the DNS and anchored forecast before and after the policy announcement of August 9, 2011, that was discussed in Figure 4. The figure shows that before the announcement the DNS and anchored yield curve forecasts similarly over-predicts the actual yield curve, whereas after the announcement the anchored forecast quickly incorporates the information contained in the FOMC statement, resulting in a sizable downward shift of the yield curve towards the actual realization. The DNS forecasts, instead, continue to largely over-predict the actual yield curve. This showcases the ability of the anchoring method to swiftly incorporate the informational advantage that surveys have about short yields and transmit it to the rest of the curve.

[Figure 5 about here.]

To understand whether the relative (point-forecast) performance of the anchored forecast and the base model changes over time, in Figure 6 we report rolling MSFEs for the two forecasts computed over rolling windows of four years, for a selection of maturities (rows) and of forecast horizons (columns). The figure shows that the anchored forecast is almost always more accurate, except for the very early years of the sample and for the 10-year maturity. Finally, note that the superior performance of the anchored forecast relative to the baseline model is not confined to the performance of the point forecast, as one implication of Proposition 1 is that, when the testable condition holds, the entire density forecast of the anchored model is more accurate than the density forecast implied by the baseline model. The results of the fluctuation test reported in Figure 2, therefore, implicitly show that anchoring using the 3-month yield provides more accurate density forecasts over time than using the base model.

[Figure 6 about here.]

5 Conclusions

We proposed a formal and computationally simple anchoring method for incorporating survey expectations into a model-based forecast of the yield curve. The method constrains the dynamics of some yields to replicate those of the survey expectations and implicitly incorporates into the forecasts of the whole yield curve any information that survey participants use without the need to explicitly model it. The method is generally applicable to a situation where one has model-based forecasts for a set of variables and extra-model forecasts for a subset of these variables. The choice of the anchors is also general, for example they could come from another model (e.g., a random walk). All it matters for whether they will deliver (density-forecast) accuracy improvements is that they satisfy the condition of Proposition 1.

We applied the method to the Dynamic Nelson and Siegel model of Diebold and Li (2006) anchored to the 3-month yield forecast from the Blue Chip Financial Analysts survey and found large and significant improvements not only in density forecast accuracy relative to the base model, but also in out-of-sample point-forecast accuracy, with typical gains of about 30% and up to 52%. Remarkably, the anchored forecast also outperformed out-of-sample a random walk benchmark, at least for short maturities and forecast horizons.

We provide an interpretation for the accuracy gains of the anchored forecasts and relate them to their ability to capture information about real economic activity as well as forward-looking information contained in monetary policy announcements. This is likely to make the method even more relevant in the future, given that several central banks such as the Federal Reserve, the European Central Bank, and the Bank of England are increasingly adopting forward guidance as a nonstandard monetary policy measure.

Finally, our method offers a way to formally incorporate into yield curve forecasts "hidden" or "unspanned" factors that go beyond the information contained in the cross-section of yields, and suggests that any successful attempt to explicitly model the dynamics of yields should acknowledge the value of forward-looking information.

There are alternative ways to incorporate survey information into yield curve models that have been considered in the literature. Advantages of the method considered in this paper are that (i) it shows that using survey forecasts as anchors improves the overall density forecasting performance of model-based forecasts, and (ii) the selection of what survey information to use is based on a formal criterion. Advantages of alternative methods are that (i) the forecasting power of surveys does not have to be directly for a yield but may also be at the level of latent factors, and (ii) it is not necessary to use a 2-step approach of first selecting a survey forecast to anchor to, and then using this anchor in the forecast.

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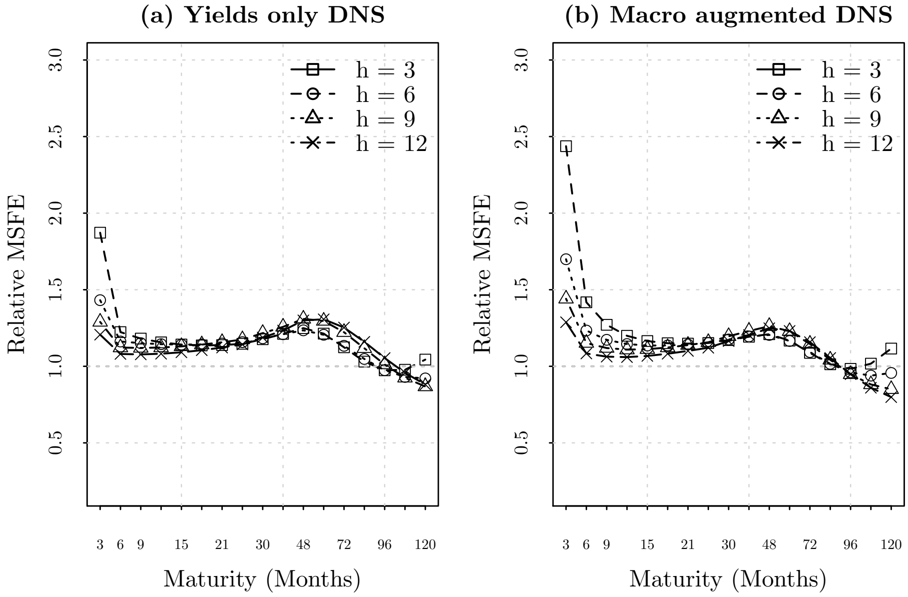
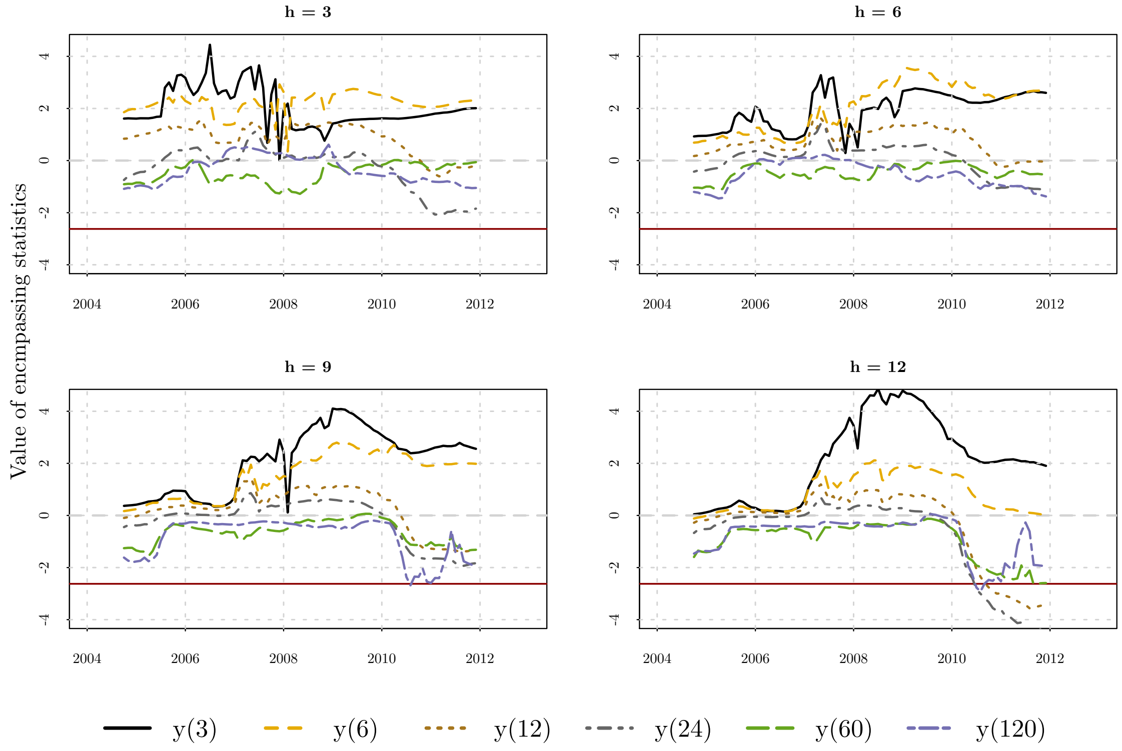
Figure 1. Relative MSFE for the yield only and macro augmented DNS against the random walk



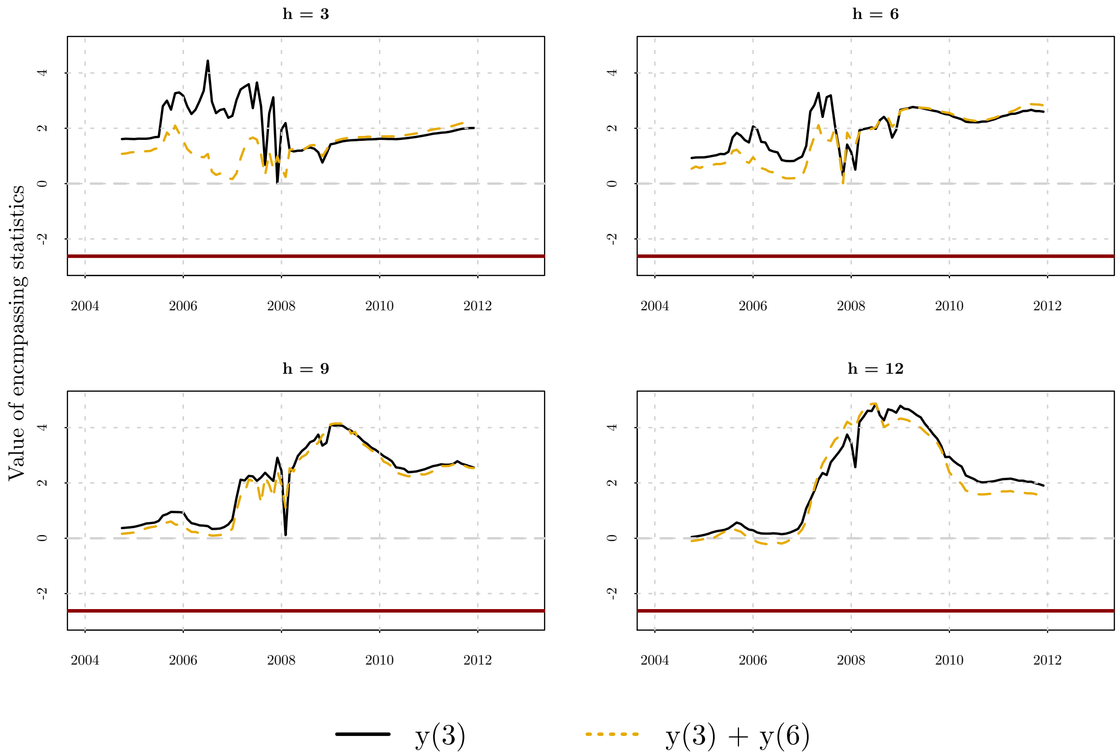
Notes: The figure reports the ratios of the MSFE for the yield only and macro augmented DNS against the random walk for different maturities and forecast horizons. Values larger than 1 indicate that the random walk outperforms the model.

Figure 2. Testing the condition in Proposition 1



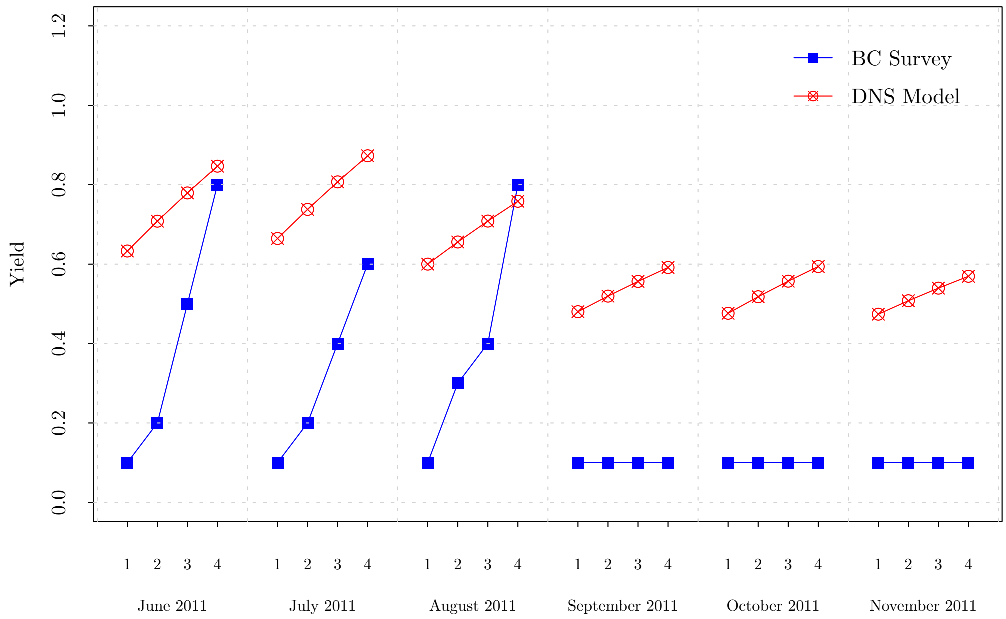
Notes: The figure reports the sequence of test statistics for testing the condition in Proposition 1, for a selection of maturities and horizons for which survey forecasts are available. The null hypothesis is rejected when the sequence of test statistics crosses the horizontal solid line, which represents the critical value (which equals -2.62 for test statistics computed over an estimation window that uses 40% of the out-of-sample observations and for a 5% significance level).

Figure 3. Testing the condition in Proposition 1



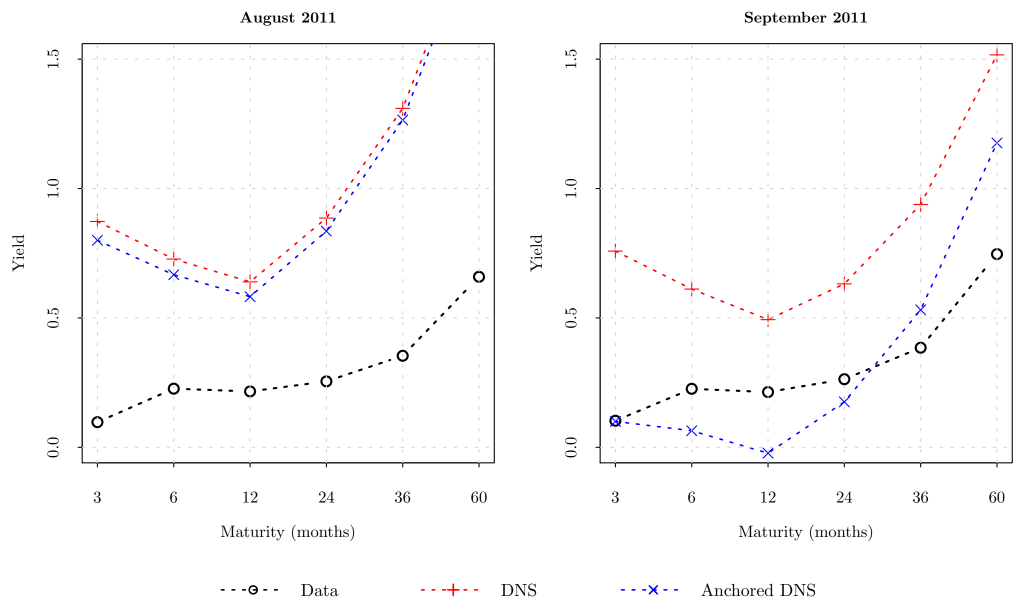
Notes: The figure reports the sequence of test statistics for testing the condition in Proposition 1, when anchoring using both the 3-month and the 6-month survey forecast. The null hypothesis is rejected when the sequence of test statistics crosses the horizontal solid line, which represents the critical value (which equals -2.62 for test statistics computed over an estimation window that uses 40% of the out-of-sample observations and for a 5% significance level).

Figure 4. The informational advantage of surveys over models



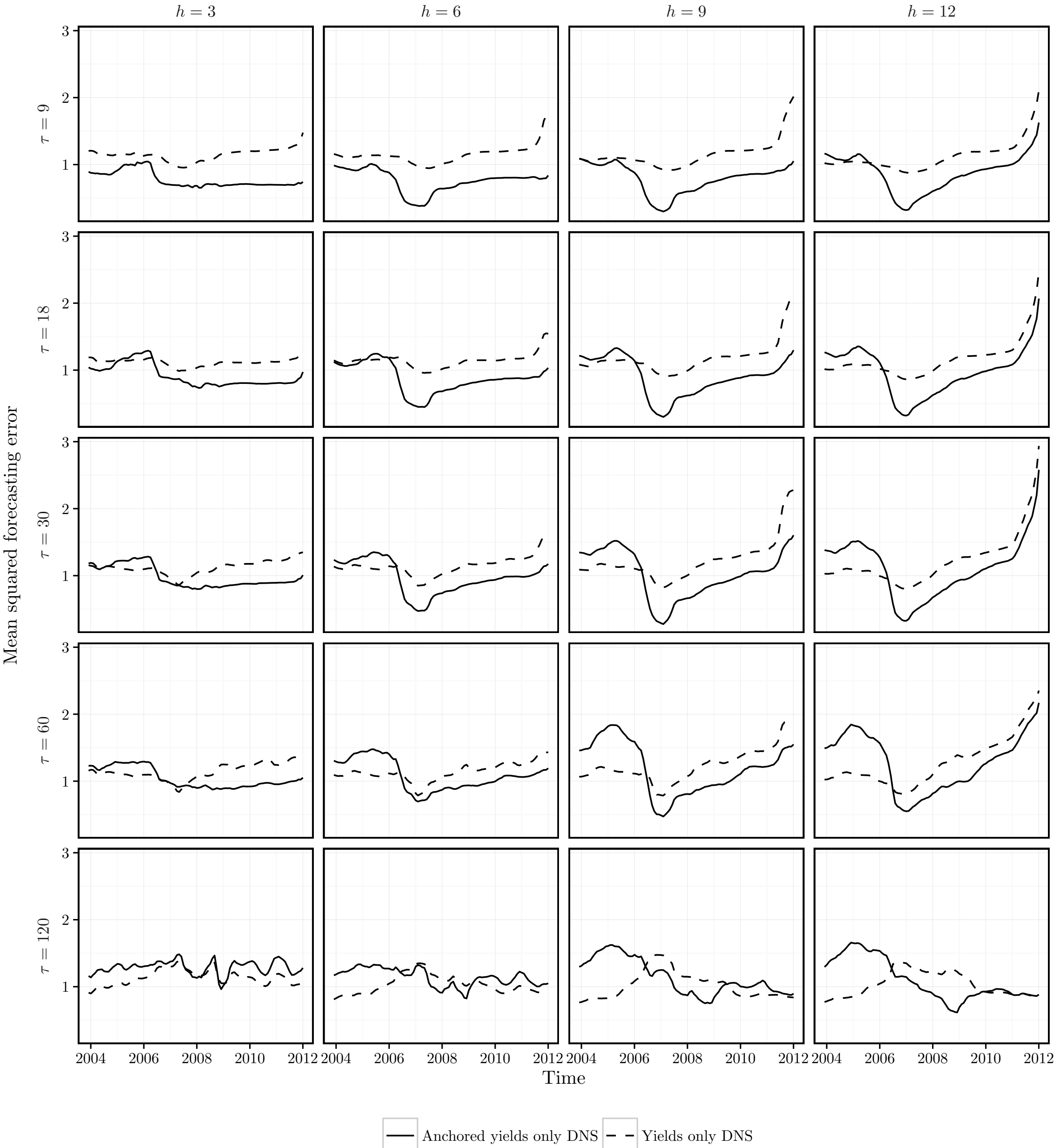
Note: The figure reports the 1- to 4-quarter-ahead forecasts of the 3-month yield given by the DNS model and the BC survey before and after the FOMC Statement of August 9, 2011.

Figure 5. DNS and Anchored forecasts before and after a monetary policy announcement



Notes: The figure shows the 12-month-ahead yield curve forecast implied by the DNS model and the corresponding anchored forecast made before and after the FOMC Statement of August 9, 2011, together with the actual yield curve realization.

Figure 6. Time-varying performance of anchored and DNS forecasts



Note: MSFE computed over a rolling four-year window

Table 1. Critical values for testing the condition in Proposition 1 ($k_{\delta,\alpha}$)

α	δ								
	0.1	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*
0.05	3.176	2.938	2.770	2.624	2.475	2.352	2.248	2.080	1.975
0.10	2.928	2.676	2.482	2.334	2.168	2.030	1.904	1.740	1.600

Table 2. Relative MSFEs of anchored forecasts

Maturity	Anchored vs DNS				Anchored vs RW			
	h=3	h=6	h=9	h=12	h=3	h=6	h=9	h=12
3	0.42***	0.58*	0.70	0.83	0.79***	0.84*	0.90	1.00
6	0.58***	0.66***	0.74*	0.84	0.71***	0.77***	0.83*	0.91
9	0.66**	0.70**	0.77	0.87	0.78**	0.81**	0.86	0.93
12	0.72*	0.74*	0.79	0.89	0.84*	0.85*	0.89	0.96
15	0.77	0.77	0.81	0.91	0.88	0.88	0.92	0.99
18	0.80	0.80	0.83	0.93	0.90	0.91	0.95	1.02
21	0.81	0.82	0.85	0.94	0.93	0.94	0.99	1.06
24	0.82	0.84	0.87	0.96	0.95	0.97	1.02	1.09
30	0.84	0.86	0.90	0.98	0.98	1.02	1.09	1.17
36	0.84	0.89	0.92	1.01	1.02	1.07	1.16	1.24
48	0.86	0.92	0.96	1.04	1.07	1.13	1.26	1.35
60	0.88	0.95	0.99	1.07	1.07	1.14	1.28	1.40
72	0.92	0.98	1.02	1.10	1.03	1.11	1.25	1.37
84	0.97	1.02	1.06	1.12	1.00	1.07	1.18	1.30
96	1.04	1.06	1.10	1.15	1.01	1.04	1.11	1.21
108	1.10	1.11	1.14	1.17	1.08	1.03	1.05	1.12
120	1.15	1.15	1.17	1.19	1.20	1.06	1.02	1.05

Notes: The table reports the ratios of MSFE for the models considered. The asterisk indicates significance according to the Diebold and Mariano (1995) test of equal accuracy against the alternative that the anchored forecast is more accurate ('***' at the 1%, '**' at the 5%, and '*' at the 1%). The test was implemented using an HAC estimator with $h-1$ as truncation parameter.

Table 3. Relative MSFEs of anchored forecasts. Restricted sample: 1980:01-2008:12.

Maturity	**Anchored vs DNS**				**Anchored vs RW**			
	h=3	h=6	h=9	h=12	h=3	h=6	h=9	h=12
3	0.47***	0.62**	0.72	0.83	0.77***	0.82**	0.88	0.96
6	0.61***	0.68***	0.75*	0.84	0.70***	0.76***	0.81*	0.88
9	0.67***	0.72**	0.78	0.86	0.76***	0.80**	0.84	0.90
12	0.71**	0.75*	0.80	0.88	0.81**	0.83*	0.87	0.92
15	0.75*	0.78	0.82	0.90	0.84*	0.86	0.89	0.94
18	0.78	0.80	0.84	0.92	0.87	0.89	0.92	0.96
21	0.80	0.82	0.86	0.93	0.89	0.91	0.94	0.99
24	0.82	0.84	0.88	0.95	0.91	0.93	0.97	1.01
30	0.84	0.87	0.91	0.98	0.95	0.98	1.03	1.06
36	0.85	0.90	0.94	1.01	0.99	1.02	1.08	1.11
48	0.87	0.93	0.99	1.05	1.04	1.08	1.16	1.20
60	0.89	0.96	1.03	1.09	1.04	1.10	1.20	1.25
72	0.92	0.99	1.08	1.13	1.01	1.09	1.20	1.26
84	0.95	1.02	1.12	1.17	0.98	1.05	1.18	1.24
96	1.00	1.06	1.17	1.21	0.96	1.02	1.14	1.20
108	1.05	1.10	1.21	1.24	1.00	1.01	1.11	1.16
120	1.09	1.13	1.25	1.26	1.09	1.03	1.10	1.13

Notes: The table reports the ratios of MSFE for the models considered. The asterisk indicates significance according to the Diebold and Mariano (1995) test of equal accuracy against the alternative that the anchored forecast is more accurate ('***' at the 1%, '**' at the 5%, and '*' at the 1%). The test was implemented using an HAC estimator with $h-1$ as truncation parameter.